

## Research Note

# Binary black holes in Mkns as sources of gravitational radiation for space based interferometers

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**Abstract.** The possibility that some Markarian objects (e.g. Mkn 501, Mkn 421 and Mkn 766) host massive binary black hole systems with eccentric orbits at their centers has been considered. These systems could be sources of gravitational radiation for space-based gravitational wave interferometers like LISA and ASTROD. In the framework of the Lincoln – Will approximation we simulate coalescence of such systems, calculate gravitational wave templates and discuss parameters of these binary black hole systems corresponding to the facilities of LISA and ASTROD. We discuss also the possibility to extract information about parameters of the binary black hole systems (masses, of components, distances between them, eccentricity and orbit inclination angle with respect to line of sight) from future gravitational wave measurements.

**Key words.** black hole physics; gravitational waves

## 1. Introduction

BL Lacertae objects, also known as Markarian objects (hereafter Mkns), belong to the class of active galaxies according to the well-established unified model on radio-loud active galactic nuclei (Urry & Padovani 1995). These objects are thought to be dominated by relativistic jets seen at small angles to the line of sight.

Until now, several astrophysical phenomena have been attributed to binary black holes, like precession of jets (Begelman, Blandford & Rees 1980), misalignment (Conway & Wroble 1995), periodic outburst activity in the Quasar OJ 287 (Sillanpää, Haarala, Valttonen et al. 1988; Lehto & Valttonen 1996) and precession of the accretion disk under gravitational torque (Katz 1997).

It has been recently observed that some Mkns show a periodic behavior in the radio, optical, X-ray and  $\gamma$ -ray light curves that is possibly related to the presence of a massive binary black hole with a jet along the line of sight or interacting with an accretion disk (Yu 2002). Therefore, the search for light curve variability, mainly in X-ray and  $\gamma$ -ray wavelengths, can be considered as a method to probe

the existence of a massive binary black hole in the center of a galaxy.

A question naturally arises: how can a binary system of massive black holes be formed? The answer to this problem can be found in the framework of the hierarchical vision of the universe (White 1997), for example if massive black hole systems form as a result of merging processes between galaxies, each of them may contain in the center a massive black hole (Rees 1984; Kormendy & Richstone 1995; Richstone, Ajhar & Bender 1998). Recent observational signatures<sup>1</sup> of such hypothesis were discussed, for example by Yu & Lu (2001) who analyzed typical features of Fe  $K_{\alpha}$  line shapes.

At least three Mkns (i.e. Mkn 501, Mkn 421 and Mkn 766) are particularly well studied at high energies, revealing a possible periodic behavior in their light curves.

Mkn 501, at  $z = 0.034$ , shows a clear well-correlated 23 day periodicity in X-ray and TeV energy bands with an observed TeV flux ratio  $f \simeq 8$  between the maximum and minimum of the signal (Protheroe, Bhat & Fleury

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<sup>1</sup> The gravitational wave spectrum from coalescing massive black hole binaries formed by merging processes of their host galaxies has been studied by Jaffe & Backer (2003).

1998; Hayashida, Hirasawa & Ishikawa 1998; Kranich, DeJager, Kestel et al. 1999; Nishikawa, Hayashi, Chamoto et al. 1999; Kranich, DeJager, Kestel et al. 2001), while evidence for correlations in the optical U-band is rather weak (Catanese, Bradbury, Beslin et al. 1997; Aharonian, Akhperjanian, Barrio et al. 1999a,b). It has been also suggested that the complex morphology of the jet and the peculiar behavior of its spectral energy distribution are probably related to the presence of a massive binary black hole (Conway & Wroble 1995; Villata & Raiteri 1999) at the center of Mkn 501.

Mkn 421, at  $z = 0.031$ , is the brightest BL Lacertae object at X-ray and UV wavelengths and it is the first extragalactic source discovered at TeV energies (Punch, Akerlof, Cawley et al. 1992). This nearby source, which has been recently observed by the XMM-Newton (Brinkmann, Sembay, Griffiths et al. 2001) and by Beppo-SAX (Maraschi, Fossati, Tavecchio et al. 1999) satellites, shows remarkable X-ray variability correlated with strong activity at TeV energies (George, Warwick & Bromage 1988) on a time-scale of  $\simeq 10^4$  s (Maraschi, Fossati, Tavecchio et al. 1999) and with a flux ratio  $f \simeq 2$ .

X-ray observations of the nearby Mkn 766, at  $z = 0.013$ , have been performed by the XMM-Newton satellite (Boller, Keil, Trümper et al. 2001). These observations have revealed the presence of a strong X-ray periodic signal with frequency  $\simeq 2.4 \times 10^{-4}$  Hz and flux ratio  $f \simeq 1.3$ .

Based on the assumption that the periodic behavior of the observed light curve for Mkn 501 is related to the presence of a binary system of black holes (one of which emits a jet moving with Lorentz factor  $\gamma_b$ ) on circular orbits, Rieger & Mannheim (2000) have proposed a method to determine the physical parameters of the binary system from the observed quantities, i. e. signal periodicity, flux ratio between maximum and minimum signal and power law spectral index. However, binary black holes might be on eccentric orbits and eccentricity values up to 0.8–0.9 are not necessarily too extreme (Fitchett 1987). This could happen frequently if binary systems of black holes at galaxy centers form generally as a consequence of merging processes between their host galaxies.

Of course, due to gravitational wave emission, orbits tend, as a first approximation, to circularize but this happens within a time-scale of the same order of magnitude as the merging time-scale (Peters 1964; Fitchett 1987). Therefore, if a massive binary black hole is found at the center of a galaxy, it may happen that the constituting black holes are still on eccentric orbits, in which case the method proposed by Rieger & Mannheim (2000) does not hold.

Hence, De Paolis, Ingrosso & Nucita (2002) studied the massive black hole binary system possible in the center of some Mkn objects by assuming more general elliptical orbits and considered the orbit eccentricity  $e$ , the binary separation  $a$  and Lorentz factor  $\gamma_b$  as free fit parameters

used to determine the masses ( $M_1$  and  $M_2$ ) of the two black holes from the observed X-ray periodicity. Once the orbital parameters of the MBH (i.e. Massive Black Hole) binaries are known, the values of the obtained orbital separation, eccentricity and MBH masses are considered as initial conditions in the time evolution of the binary systems.

In the present paper we study the evolution of the system and to determine the gravitational wave waveforms, i.e. the amplitude of the metric perturbation as a function of time. In doing this, we simulate the evolution of binary black hole systems by using the Lincoln & Will (1990) approximation and calculate gravitational wave templates without any assumptions about the evolution of our system on quasi-circular orbits. We also study the detectability of the emitted gravitational waves by the next generation of space-based interferometers like LISA (Reinhard 2000) and ASTROD (Wu, Xu & Ni 2000; Ni 2002).

The paper is structured as follows: in Section 2 we show how to determine the massive black hole binary system parameters starting from the observed X-ray periodicity toward the considered Mkn objects. In Section 3, the model we use to simulate the binary system evolution in the post<sup>5/2</sup>-Newtonian approximation is reviewed. In Section 4 we describe our results about the evolution of binary system, profile gravitational wave templates, typical times for the evolution of our binary systems before fitting into the LISA frequency band. Finally, in Section 5 we draw some conclusions.

## 2. Initial conditions for a coalescing binary massive black hole system

As discussed above, there exist a number of Mkn objects (like Mkn 501, Mkn 421 and Mkn 766) that show periodic activity in the radio, optical, X-ray and  $\gamma$ -ray light curves. It has been recently proposed that the observed periodicity is possibly related to the presence of a massive binary black hole creating a jet along the line of sight.

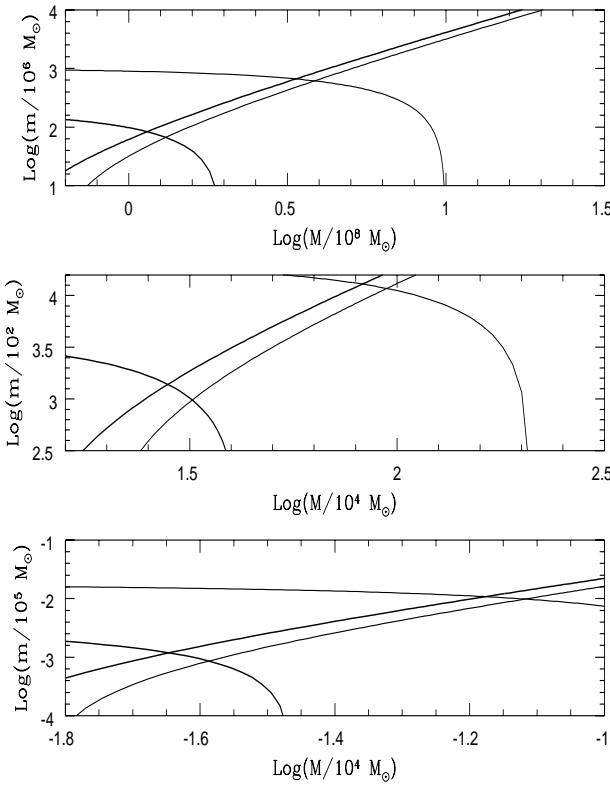
Following Rieger & Mannheim (2000), we assume that the observed signal periodicity has a geometrical origin, being a consequence of a Doppler-shifted modulation. It is therefore possible to relate the observed signal period  $P_{\text{obs}}$  to the Keplerian orbital period  $P_k$  by

$$P_{\text{obs}} = (1 + z) \left( 1 - \frac{v_z}{c} \cos i \right) P_k , \quad (1)$$

where  $i \simeq 1/\gamma_b$  and  $v_z$  is the typical jet velocity assumed to be  $v_z \simeq c(1 - 1/\gamma_b^2)^{1/2}$  (Spada 1999). Here, we rely on the assumption that the lighter black hole in the binary system is emitting a jet which is moving toward the observer with Lorentz factor  $\gamma_b$ .

The observed flux modulation due to Doppler boosting can be written as

$$S(\nu) = \delta^{3+\alpha} S'(\nu) , \quad (2)$$



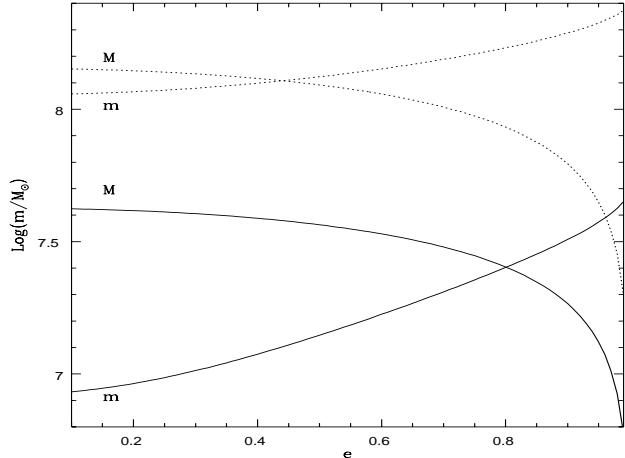
**Fig. 1.** The secondary mass  $m$  (in the text  $M_1$ ) is shown as a function of the primary mass  $M$  (in the text  $M_2$ ) for the black hole binaries at the center of Mkn 501, Mkn 421 and Mkn 766 (from the upper to the bottom panel). The thick and thin lines represent the conditions expressed in eqs. (7) with Lorentz factor  $\gamma_b = 10$  and  $\gamma_b = 15$ , respectively. The binary semi-major axes  $a$  are set to  $5 \times 10^{16}$  cm,  $2 \times 10^{14}$  cm and  $1 \times 10^{13}$  cm for Mkn 501, Mkn 421 and Mkn 766. The orbital eccentricity in all cases is set equal to 0.5. The intersection between lines corresponding to the same Lorentz factor gives the masses of the black holes in the considered binary system.

where  $\alpha$  is the source spectral index<sup>2</sup> and the Doppler factor  $\delta$  is given by

$$\delta = \frac{\sqrt{1 - (v_z^2 + v_{ls}^2)/c^2}}{1 - (v_z \cos i + v_{ls} \sin i)/c} . \quad (3)$$

Here,  $v_z$  is the jet velocity,  $i$  is the inclination angle between the jet axis and the line of sight and  $v_{ls}$  the component of the less massive black hole velocity along the line of sight.

<sup>2</sup> Values of the power law index  $\alpha$  are found to be 1.2, 1.7 and 2.11 for Mkn 501, Mkn 421 and Mkn 766, respectively. For more details see Rieger & Mannheim (2000), Guainazzi, Vacanti, Malizia et al. (1999) and Boller, Keil, Trümper et al. (2001).



**Fig. 2.** Primary and secondary black hole masses are plotted as a function of the orbit eccentricity  $e$  for the binary system in Mkn 501. Dotted lines correspond to Lorentz factor  $\gamma_b = 10$  while solid lines correspond to  $\gamma_b = 30$ . The semi-major axes is  $a = 8 \times 10^{16}$  cm. Similar plots can be obtained for the Mkn 421 and Mkn 766.

Depending on the position of  $M_1$  along its orbit, the velocity  $v_{ls}$  ranges between a minimum and a maximum value corresponding, through eq. (3), to the two extremal values for the Doppler factor given by

$$\delta_{\max} = \frac{\sqrt{1 - \frac{(v_z^2 + v_{ls}^2)}{c^2}}}{1 - \left(v_z \cos i + \frac{2\pi R_a \sin i}{P_k(1-e^2)^{1/2}}\right) c^{-1}} ,$$

$$\delta_{\min} = \frac{\sqrt{1 - \frac{(v_z^2 + v_{ls}^2)}{c^2}}}{1 - \left(v_z \cos i - \frac{2\pi R_a \sin i}{P_k(1-e^2)^{1/2}}\right) c^{-1}} , \quad (4)$$

where  $R_a = M_2 a / (M_1 + M_2)$ , being  $a$  and  $e$  the binary semi-major axis and the orbit eccentricity, respectively. From eq. (2), the observed maximum and minimum fluxes modulated by the Doppler effect turn out to be

$$S_{\max}(\nu) = \delta_{\max}^{3+\alpha} S'(\nu) , \quad (5)$$

$$S_{\min}(\nu) = \delta_{\min}^{3+\alpha} S'(\nu) ,$$

so that one obtains the condition  $\delta_{\max}/\delta_{\min} \simeq f^{1/(3+\alpha)}$ , where  $f = S_{\max}(\nu)/S_{\min}(\nu)$  is the observed maximum to minimum flux ratio.

By using eqs. (5) and (6) we have

$$R_a = \frac{c P_k f^{1/3+\alpha} - 1}{2\pi f^{1/3+\alpha} + 1} \left( \frac{1}{\sin i} - \frac{v_z}{c} \cot i \right) (1 - e^2)^{\frac{1}{2}} . \quad (6)$$

Finally, eqs. (1) and (6) can be rewritten in the following form (for more details see

De Paolis, Ingrosso & Nucita (2002))

$$\left\{ \begin{array}{l} \frac{M_2}{(M_1 + M_2)^{2/3}} = \frac{P_{\text{obs}}^{1/3}}{[2\pi(1+z)G]^{1/3} \sin i} \times \\ \quad \frac{f^{1/3+\alpha} - 1}{f^{1/3+\alpha} + 1} \left(1 - \frac{v_z}{c} \cos i\right)^{2/3} (1 - e^2)^{1/2}, \\ M_1 + M_2 = \left[ \frac{2\pi(1+z)}{P_{\text{obs}}} \left(1 - \frac{v_z}{c} \cos i\right) \right]^2 \frac{a^3}{G}. \end{array} \right. \quad (7)$$

In these equations  $\gamma_b$ ,  $e$  and  $a$  have to be considered as free model parameters to be determined by the observed data ( $P_{\text{obs}}$ ,  $f$  and  $\alpha$ ) of the three Mkn objects of interest. In this way, we are able to estimate the masses of the black holes supposed to be composing each of the MBH binaries. For the three considered Mkn objects, the secondary mass  $M_1$  as a function of the primary one  $M_2$  is shown in Fig. 1. The intersection between lines corresponding to the same Lorentz factor ( $\gamma_b = 10$  for the thick lines and  $\gamma_b = 15$  for the thin lines) gives the masses of the black holes in the considered binary system.

In Fig. 2 (for Mkn 501) the primary and secondary black hole masses as a function of the orbit eccentricity ( $\gamma_b = 10, 30$  and  $a = 8 \times 10^{16}$  cm) are shown. In this case, the black hole masses are in the range  $10^6 - 10^9 M_\odot$ .

The previous method allows us to determine the orbital parameters of the massive binary black hole possibly in the center of some Mkn objects. The obtained values for the semi-major axes  $a$  and eccentricity  $e$  obviously changes in time as a consequence of emission of gravitational radiation. However, as shown by Lincoln & Will (1990), when studying binary systems non-negligible relativistic corrections in the equations of motion have to be considered. It follows that in the simulation of the evolution of the binary systems, the obtained values for the semi-major axes and eccentricity have to be considered as the initial conditions of the problem, i.e.  $a_i$  and  $e_i$ , respectively.

### 3. Model for a coalescing binary system evolution

To analyze the binary black hole evolution we use a (post) $^{5/2}$ -Newtonian approximation developed by Lincoln & Will (1990).<sup>3</sup>

Earlier, Grishchuk & Kopejkin (1986) derived similar equations of motion for isolated bodies in a (post) $^{5/2}$ -Newtonian approximation taking into account the effects of gravitational radiation friction which lead to orbital shrink. In studying these effects Grishchuk & Kopejkin (1986) derived their equations using osculating elements like those reported in Lincoln & Will (1990). Here we use the Lincoln & Will (1990) approach since it is very convenient for our computational aims. Of course, there are a

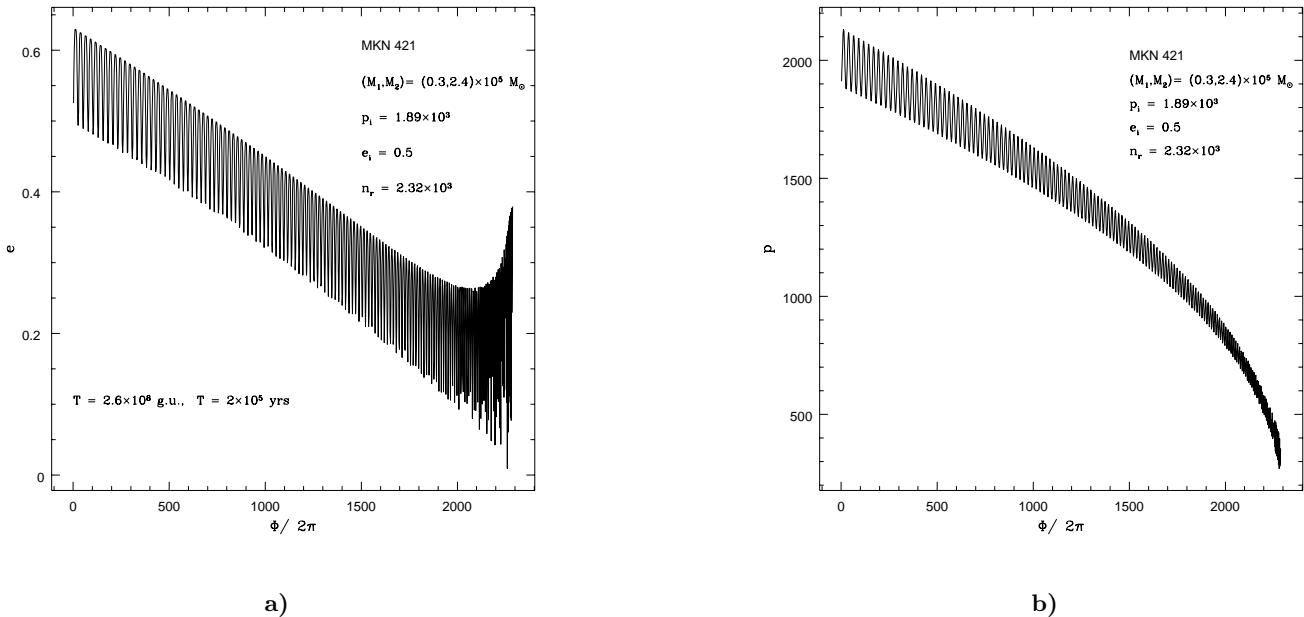
<sup>3</sup> One can see also Zakharov (1996, 1997, 1998) where this approximation was used to analyze the evolution of a binary neutron star system in which the lower mass object is losing matter since it fills its Roche lobe.

lot of other more precise approaches which may allow one to describe the final stages of evolution of binary black hole systems (see, for example, Buonanno & Damour (1999, 2000); Fiziev & Todorov (2001); Damour (2001); Buonanno (2002); Mora & Will (2002); Gourghoulhon, Grandclement & Bonazzola (2002a,b); O'Shaughnessy (2003); Buonanno, Chen & Valisneri (2003) and references therein). Even the transition from inspiral stage to plunge for circular orbits was discussed by Ori & Thorne (2000) and for eccentric orbits by O'Shaughnessy (2003). Glampedakis & Kennefick (2002); Glampedakis, Hughes & Kennefick (2002) used a Kerr metric approximation to analyze the evolution of a binary black hole with a small mass ratio, namely  $m/M \sim 10^{-4} - 10^{-6}$ , where  $m$  and  $M$  are the masses of the captured body and the central black hole, respectively. Rapidly spinning massive black hole binary systems as possible sources for LISA were considered by Vecchio (2003) in the framework of the post $^{1.5}$ -Newtonian approximation and it was found that the black hole spin could be a very essential factor and could drastically change the signature of the gravitational wave signal. However, in this paper we decided to use the Lincoln – Will approximation since, as it was mentioned before,  $5/2$  is the first order of the post-Newtonian approximation where one could introduce the gravitational radiation friction in a self-consistent way.

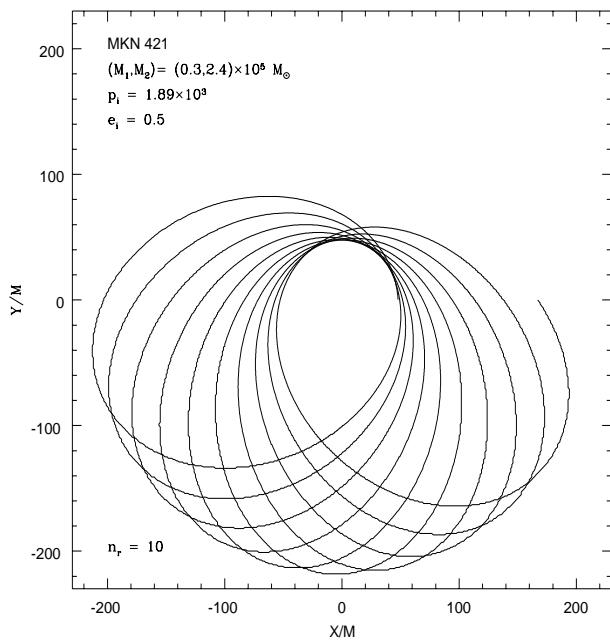
Moreover, usually these binary black hole systems are so far from the plunge stage of their evolution that it can be simulated by using the lowest self-consistent post-Newtonian approximation. Thus, one could use more simple treatments like that proposed by Peters & Mathews (1963); Peters (1964). However, in the framework of these approximations, the motion and the time evolution of the binary system cannot be studied in detail since some relativistic effects, such as the “perihelion shift” (see, for example, Ehlers et al. (1976); Grishchuk & Kopejkin (1986) for details), are not accounted for. In addition, Ehlers et al. (1976) have some doubts about the accuracy of such methods since approaches similar to the Peters & Mathews (1963) and Peters (1964) approximations are mathematically inconsistent and may lead to errors of the same order as the effects being considered. Thus, for a more accurate treatment of the problem we follow the Lincoln & Will (1990) approach, to which we refer for more details.

The orbital motion of two massive black holes moving around the common center of mass is strongly influenced by gravitational radiation losses. Viewing this as a Newtonian orbital motion with perturbation suggests use of the osculating orbital element method taken from celestial mechanics.

The basic scenario is the following: at any time one can find a Keplerian orbit which is tangent to the true orbit. This means that both the position and the velocity of the particle on the true orbit coincide with the position and the velocity of the tangent Keplerian orbit at the considered time. Of course, at a subsequent instant the actual



**Fig. 3.** For Mkn 421, the evolution of a MBH binary system is shown. The orbital parameters reported in each panel allow to reproduce the X-ray light curve periodicity. The evolutions of both the MBH binary system eccentricity  $a$ ) and separation  $b$ ) are given as a function of the number of revolutions  $\Phi/2\pi$ . The initial parameter  $p_i = a_i(1 - e_i^2)$  is given in unit of  $GM/c^2$  being  $M = M_1 + M_2$ .

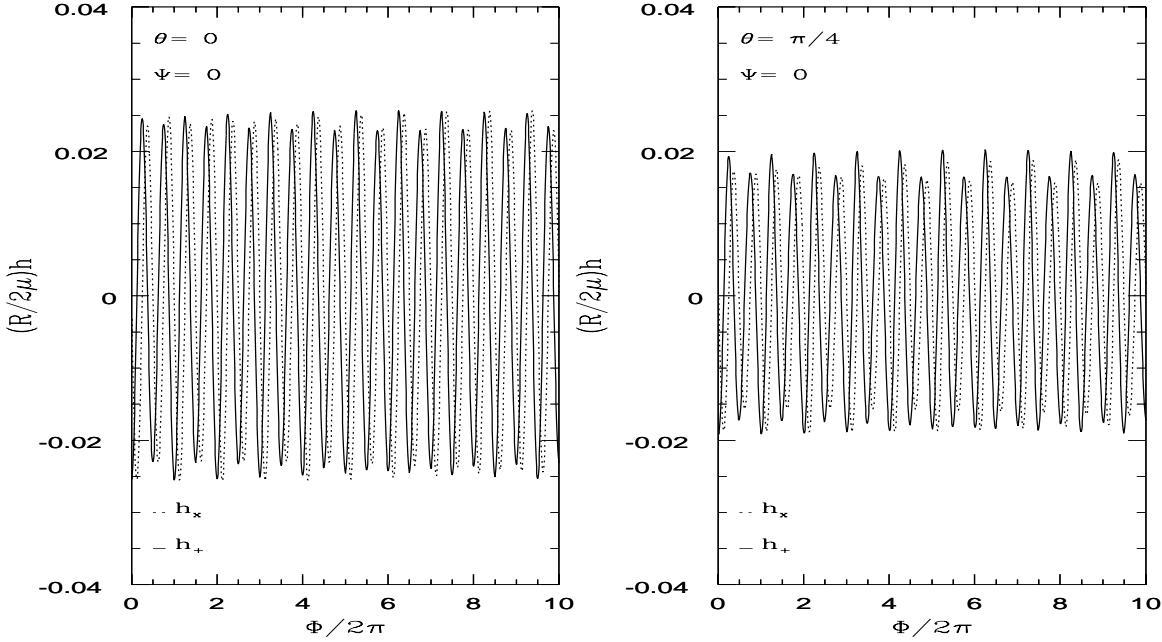


**Fig. 4.** The orbital evolution of the black hole binary at the center of Mkn 421 with initial parameters able to reproduce the observed lightcurve periodicity is shown. The orbital perihelion shift is evident.

orbit will be tangent to a different Keplerian orbit. In the osculating orbit formalism a general two-body orbit is gen-

erally described by six parameters:  $i$  the inclination of the orbit with respect to a reference plane,  $\Omega$  the line of the ascending node,  $\omega$  the angle between the line of node and the pericentric line,  $a$  the semi-major axis,  $e$  the orbital eccentricity and  $T$  the time of pericentric passage. According to Lincoln & Will (1990), these quantities are coupled by a set of first-order differential equations, see eq. (2.11a)-(2.11c) in the previously mentioned paper, which can be numerically solved, by appropriately choosing the initial conditions, obtaining  $a = a(\Phi)$ ,  $e = e(\Phi)$  and  $\omega = \omega(\Phi)$ ,  $\Phi$  being the usual polar angle which in turn depends on time  $t$  through the relation  $r^2\dot{\Phi} = (Mp)^{1/2}$ . Here, the auxiliary parameter  $P$  is given by  $p = a(1 - e^2)$ .

Applying this approach, we are able to describe the motion and the orbital evolution of the binary system. In Figs. 3 *a*) and *b*) the evolution of the possible Mkn 421 MBH binary system is shown. Notice that the orbital parameters reported in the same panels correspond to a set of values of the Lorentz factor  $\gamma_b$ , semi-major axis  $a$  and orbital eccentricity  $e$  reproducing the observed X-ray light curve periodicity. The evolution of both the MBH binary system eccentricity *a*) and separation *b*) are given as a function of the number of revolutions  $\Phi/2\pi$ . In the same panels the expected coalescing time, calculated as in Lincoln & Will (1990), is also reported both in geometrical units ( $2.6 \times 10^8$ ) and in years ( $2 \times 10^5$  yrs) for the considered MBH binary system. For comparison, the coalescing time scale of an eccentric binary calculated using the Peters approximation (see eq. (5.14) in Peters (1964)) gives  $\simeq 1.3 \times 10^5$  yrs. It is not surprising that the two coalescing time scales differ, since in Lincoln & Will



**Fig. 5.** The two polarization states ( $h_x$  and  $h_+$ ) of the gravitational waves emitted by the MBH binary system (with reduced mass  $\mu$ ), at distance  $R$  from Earth, for the selected model in Fig. 3 are shown. The waveforms are given for the first 10 orbital revolutions. The two parameters  $\Theta$  and  $\Psi$  determine the direction of observation. In principle, the study of the gravitational wave templates will allow one to extract information about the MBH orbital parameters.

(1990) different approximations and relativistic effects (as the perihelion shift) are taken into account. As evident, the difference between the time scales evaluated by using these methods is not dramatic, so that the Peters (1964) formalism can be used, as a first approximation, to evaluate the coalescing time for binaries that are (like ours) far enough from the plunge stage. A further effect that can be accurately described by using the Lincoln & Will (1990) treatment is the orbital evolution of the black hole binary system. For example in Fig. 4 we show the orbital evolution of the black hole binary in Mkn 421 with initial parameters chosen in order to reproduce the observed lightcurve periodicity. The orbital perihelion shift is clearly evident.

Once the orbital motion of the binary system is known as a function of time, the osculating orbital parameter formalism allows us to determine the gravitational wave form, i.e. the evolution in time of the metric perturbation. In particular, using eqs. (4.1a)-(4.1b) in Lincoln & Will (1990), we can evaluate the polarization states  $h_x$  and  $h_+$  of the gravitational wave emitted by the considered MBH binary system. In Fig. 5 we plot, for the first 10 orbital revolutions, the expected wave forms depending on

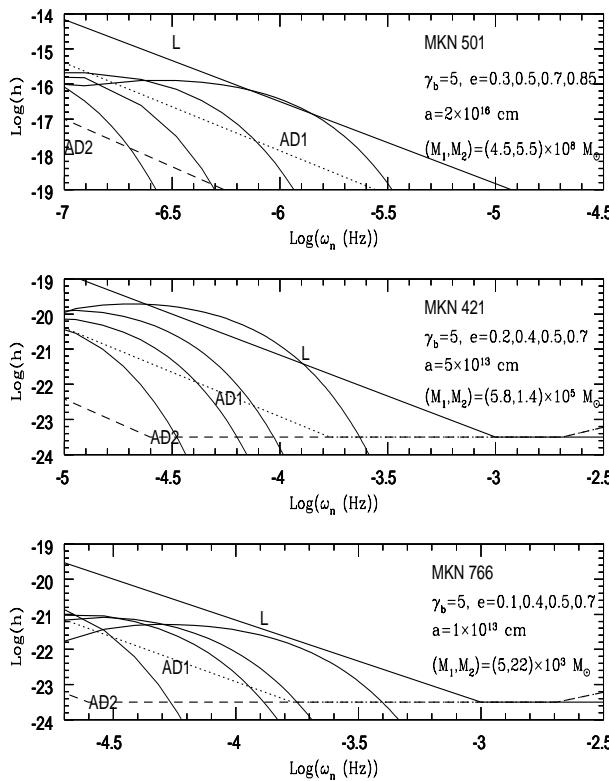
the  $\Theta$  and  $\Psi$  parameters which determine the observation direction.

#### 4. Discussion and conclusions

Usually, at the first stage of evolution, our binary systems are outside of the LISA frequency band since the typical frequency of the emitted gravitational radiation is much lower than  $10^{-4}$  Hz. However, other experiments, such as the ASTROD gravitational wave detector (Wu, Xu & Ni 2000; Ni 2002), will have a much higher sensitivity than LISA. In fact, even using the same laser power as for LISA, the ASTROD sensitivity would be shifted to a frequency lower by a factor up to 60 (30 on average), Ni (2002). Moreover, as noted by Rüdiger (2002), if the LISA accelerometer noise goal will be obtained, the ASTROD sensitivity at low frequencies will be about 30 times better than that of LISA, as indicated in Fig. 8 of Ni (2002).

<sup>4</sup> Fig. 6 shows the possibility of detecting gravitational waves from such systems using LISA and ASTROD(1) and ASTROD(2) (the detailed description of these ASTROD facilities was given by Ni (2002)). Moreover, if the absolute metrological accelerometer/intertial sensor can be

<sup>4</sup> See also Fig. 13 from the paper by Rüdiger (2002).



**Fig. 6.** Assuming particular values of the free model parameters ( $\gamma_b$ ,  $e$  and  $a$ ), we compare the expected gravitational wave spectra with the LISA (L), ASTROD1 (AD1) and ASTROD2 (AD2) thresholds, respectively. Note that the values of the black holes reported for each Mkn objects strongly depend on the chosen parameters.

developed, there is even the possibility of reaching the ASTROD(3) sensitivity curve with a shifting factor that could reach  $10^3 - 10^4$  lower than LISA (see again Fig. 8 from the paper by Ni (2002)). Note that ways to decrease the instrumental noise up to  $(1 - 3) \times 10^{-5}$  are discussed by Hughes (2002) (see also Larson, Hiscock & Hellings (2002)).

In principle there is even a non-negligible chance to determine the inclination angle for a binary black hole system using gravitational wave observations if we will have a possibility to distinguish these templates for different  $\Theta$  and  $\Psi$  angles. Of course, it could be only a hypothetical chance to extract this information from observations because one should collect data for some years to reach the necessary sensitivity to detect the emitted gravitational waves. The dependence of gravitational wave templates on  $\Theta$  and  $\Psi$  angles could be important to construct optimal filters for gravitational wave detection.

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